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Approximate Controllability of Coupled Nonlocal Partial Functional Integro-Differential Equations with Impulsive Effects

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Abstract :

In this work, we study the approximate controllability problem for a system of nonlocal integro-differential equations with impulsive effects. We start by investigating the existence and uniqueness of solutions for this system. The results are derived using the theory of resolvent operators combined with fixed point theory in a generalized Banach space. Next, we examine approximate controllability without necessarily requiring the nonlinear terms to be uniformly bounded. In particular, we do not impose here the compactness condition for either the resolvent operator or the state-dependent function in the nonlocal condition, as is commonly found in the literature. Finally, we provide an example to demonstrate the abstract results of this work.

Keywords : Integro-differential systems, resolvent operator, fractional power operators, nonlocal conditions, approximate controllability, generalized measures of noncompactness.

Mathematics Subject Classification : 93B05; 34G20; 34K30; 34K10.

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